Exercise 12

A particle is moving along a hyperbola $xy = 8$. As it reaches the point $(4, 2)$, the y-coordinate is decreasing at a rate of 3 cm/s. How fast is the x-coordinate of the point changing at that instant?

Solution

The aim here is to find dx/dt, the rate at which the x-coordinate is changing, at the point $(4, 2)$. Differentiate both sides of the given equation with respect to t and use the product rule.

$$\frac{d}{dt}(xy) = \frac{d}{dt}(8)$$

$$\frac{dx}{dt}y + x\frac{dy}{dt} = 0$$

Solve for dx/dt.

$$\frac{dx}{dt} = -\frac{x}{y}\frac{dy}{dt}$$

The y-coordinate is decreasing at a rate of 3 cm/s, so $dy/dt = -3$ cm/s. If $x = 4$ cm and $y = 2$ cm, then

$$\left.\frac{dx}{dt}\right|_{\substack{x=4\\y=2}} = -\frac{4\text{ cm}}{2\text{ cm}}\left(-3\,\frac{\text{cm}}{\text{s}}\right) = 6\,\frac{\text{cm}}{\text{s}}.$$

Therefore, the x-coordinate is increasing at a rate of 6 cm/s.